14.5 Videos Guide

14.5a

- The Chain Rule
  - For $z = f(x, y)$, where $x = x(t)$ and $y = y(t)$, $\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$
    - Exercise:
      Use the Chain Rule to find dz/dt.
      $z = \frac{x-y}{x+2y}, \quad x = e^{\pi t}, \quad y = e^{-\pi t}$

14.5b

- For $z = f(x, y)$, where $x = x(s, t)$ and $y = y(s, t)$,
  $\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s}$ and $\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}$

Exercises:

- Use the Chain Rule to find $\partial z/\partial s$ and $\partial z/\partial t$.
  $z = \tan^{-1}(x^2 + y^2), \quad x = s \ln t, \quad y = te^s$

14.5c

- Use the Chain Rule to find the indicated partial derivatives.
  $T = \frac{v}{2u+v}, \quad u = pq\sqrt{r}, \quad v = p\sqrt{q}r;$
  $\frac{\partial T}{\partial p}, \frac{\partial T}{\partial q}, \frac{\partial T}{\partial r}$ when $p = 2, q = 1, r = 4$

14.5d

- Higher-order derivatives

14.5e

- Implicit differentiation
  - If $F(x, y) = 0$, then $\frac{dy}{dx} = -\frac{F_x}{F_y}$, provided $F_y \neq 0$
  - If $F(x, y, z) = 0$, then $\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}$ and $\frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$, provided $F_z \neq 0$

Exercise:

- Use the above formula to find $\partial z/\partial x$ and $\partial z/\partial y$ for $x^2 - y^2 + z^2 - 2z = 4$.